Order-parameter model for unstable multilane traffic flow

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We discuss a phenomenological approach to the description of unstable vehicle motion on multilane highways that explains in a simple way the observed sequence of the ''free flow ↔ synchronized mode ↔ jam'' phase transitions as well as the hysteresis in these transitions. We introduce a variable called an order parameter that accounts for possible correlations in the vehicle motion at different lanes. So, it is principally due to the ''many-body'' effects in the car interaction in contrast to such variables as the mean car density and velocity being actually the zeroth and first moments of the ''one-particle'' distribution function. Therefore, we regard the order parameter as an additional independent state variable of traffic flow. We assume that these correlations are due to a small group of ''fast'' drivers and by taking into account the general properties of the driver behavior we formulate a governing equation for the order parameter. In this context we analyze the instability of homogeneous traffic flow that manifested itself in the above-mentioned phase transitions and gave rise to the hysteresis in both of them. Besides, the jam is characterized by the vehicle flows at different lanes which are independent of one another. We specify a certain simplified model in order to study the general features of the car cluster self-formation under the ''free flow ↔ synchronized motion'' phase transition. In particular, we show that the main local parameters of the developed cluster are determined by the state characteristics of vehicle motion only.

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I. INTRODUCTION. MACROSCOPIC MODELS FOR MULTILANE TRAFFIC DYNAMICS

The study of traffic flow actually formed a novel branch of physics since the pioneering works by Lighthill and Whitham [1], Richards [2], and, then, by Prigogine and Herman [3]. It is singled out by the fact that in spite of *motivated*, i.e., a nonphysical individual behavior of moving vehicles (they make up a so-called ensemble of ''self-driven particles,'' see, e.g., [4–6]), traffic flow exhibits a wide class of critical and self-organization phenomena met in physical systems (for a review see [7–9]). Besides, the methods of statistical physics turn out to be a useful basis for the theoretical description of traffic dynamics [10].

The existence of a basic phase in vehicle flow on multilane highways called synchronized motion was recently discovered by Kerner and Rehborn [11], impacting significantly the physics of traffic as a whole. In particular, it turns out that the spontaneous formation of moving jams on highways proceeds mainly through a sequence of two transitions: ''free flow → synchronized motion → stop-and-go pattern'' [12]. All the three traffic modes are phase states, meaning they have the ability to persist individually for a long time. Besides, the two transitions exhibit hysteresis [12–14], i.e., for example, the transition from the free flow to the synchronized mode occurs at a higher density and lower velocity than the inverse one. As follows from the experimental data [11,13,14] the ''free flow ↔ synchronized mode'' phase transition is essentially a multilane effect. Recently Kerner [7–9] assumed it to be caused by a ''Z''-like form of the overtaking probability depending on the car density.

The synchronized mode is characterized by substantial correlations in the car motion along different lanes because of the lane changing maneuvers. So, to describe such phenomena a multilane traffic theory is required. There have been proposed several macroscopic models dealing with multilane traffic flow and based on the gas-kinetic theory [15–20], a compressible fluid model [21] generalizing the approach by Kerner and Konhäuser [22,23], and actually a model [24,25] dealing with the time-dependent Ginzburg-Landau equation.

All these models describe traffic flow in terms of the car density ρ, mean velocity v, and, maybe, the velocity variance θ or we ascribe these quantities to vehicle flow at each lane α individually. In other words, the quantities $\{\rho,v,\theta\}_\alpha$ are regarded as a complete set of the traffic flow state variables and if they are fixed then all the vehicle flow characteristics should be determined. The given models relate the self-organization phenomena actually to the vehicle flow instability caused by the delay in the driver response to changes in the motion of the nearest cars ahead. In fact, let us briefly consider their simplified version (cf. [26,27]) which, nevertheless, catches the basic features taken into account,

$$\frac{\partial \rho}{\partial t}+\frac{\partial(\rho v)}{\partial x}=0, \tag{1}$$

$$\frac{\partial v}{\partial t}+v\frac{\partial v}{\partial x}=-\frac{1}{\rho}\frac{\partial \mathcal{P}}{\partial x}+\frac{1}{\tau'}(\mathcal{U}-v). \tag{2}$$

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Here the former term on the right-hand side of Eq. (2), a so-called pressure term, reflects dispersion effects due to the finite velocity variance θ of the vehicles and the latter one describes the relaxation of the current velocity within the time τ' to a certain equilibrium value $\mathcal{U}\{\rho,\theta\}$. In particular, for

$$\mathcal{P}\{\rho,v,\theta\}=\rho\theta-\eta\frac{\partial v}{\partial x}, \tag{3}$$

where η is a ''viscosity'' coefficient and the velocity variance θ is treated as a constant, we are confronted with the Kerner-Konhäuser model [22,23]. In the present form the relaxation time τ' characterizes the acceleration capability of the mean vehicle as well as the delay in the driver control over the headway (see, e.g., [28–30]). The value of the relaxation time is typically estimated as $\tau'\sim 30$ s because it is mainly determined by the mean time of vehicle acceleration which is a slower process than the vehicle deceleration or the driver reaction to changes in the headway. The equilibrium velocity $\mathcal{U}\{\rho\}$ (here the fixed velocity variance θ is not directly shown in the argument list) is chosen by drivers keeping in mind the safety, the readiness for risk, and the legal traffic regulations. For homogeneous traffic flow the equilibrium velocity $\mathcal{U}\{\rho\}=\vartheta(\rho)$ is regarded as a certain phenomenological function meeting the conditions

$$\frac{d\vartheta(\rho)}{d\rho}<0 \quad \text{and} \quad \rho\vartheta(\rho)\to 0 \quad \text{as} \quad \rho\to\rho_0, \tag{4}$$

where ρ_0 is the upper limit of the vehicle density on the road. Since the drivers anticipate risky maneuvers of distant cars also, the dependence $\mathcal{U}\{\rho\}$ is nonlocal. In particular, it is reasonable to assume that the driver behavior is mainly governed by the car density ρ at a certain distant ''interaction point'' $x_a=x+L^*$ rather than at the current one x, which gives rise to a new term in Eq. (2) based on the gas-kinetic theory [16,17]. Here, for the sake of simplicity following [26], we take this effect into account by expanding $\rho(x+L^*)$ and then $\mathcal{U}\{\rho\}$ into the Taylor series and we write

$$\mathcal{U}\{\rho\}=\vartheta(\rho)-v_0\frac{L^*}{\rho}\frac{\partial\rho}{\partial x}, \tag{5}$$

where v_0 is a certain characteristic velocity of the vehicles. Then linearizing the obtained system of equations with respect to the small perturbations $\delta\rho,\delta v\propto\exp(\gamma t+ikx)$ we obtain that the long-wave instability will occur if (cf. [22,23,26])

$$\tau'(\rho\vartheta'_\rho)^2>v_0L^*+\tau'\theta. \tag{6}$$

In the long-wave limit the instability increment Re γ depends on k as

$$\text{Re}\,\gamma=k^2[\tau'(\rho\vartheta'_\rho)^2-(v_0L^*+\tau'\theta)], \tag{7}$$

and the upper boundary $k_{\max}$ of the instability region in the k space is given by the expression

$$k^2_{\max}=\frac{\rho}{\tau\eta}\left\{\left[\frac{\tau'(\rho\vartheta'_\rho)^2}{(v_0L^*+\tau'\theta)}\right]^{1/2}-1\right\}$$

[here $\vartheta'_\rho=d\vartheta(\rho)/d\rho$]. As follows from Eq. (6) the instability can occur when the delay time τ' exceeds a certain critical value τ_c and for $\tau'<\tau_c$ the homogeneous traffic flow is stable at least with respect to small perturbations. Moreover, the instability increment attains its maximum at $k\sim k_{\max}$, so, special conditions are required for a wide vehicle cluster to form [22,23,31,32]. In particular, in the formal limit $\tau'\to 0$ from Eq. (2) we get

$$v=\vartheta(\rho)-\frac{D}{\rho}\frac{\partial\rho}{\partial x}, \tag{8}$$

where $D=v_0L^*$ plays the role of the diffusion coefficient of vehicle perturbations. The substitution of Eq. (8) into Eq. (1) yields the Burgers equation

$$\frac{\partial\rho}{\partial t}+\frac{\partial[\rho\vartheta(\rho)]}{\partial x}=D\frac{\partial^2\rho}{\partial x^2}, \tag{9}$$

which describes the vehicle flux stable, at least with respect to small perturbations in the vehicle density ρ.

However, the recent experimental data [11–14,33] about traffic flow on German highways have demonstrated that the characteristics of the multilane vehicle motion are more complex (for a review see [7–9]). In particular, there are actually three types of synchronized modes, the totally homogeneous state, the homogeneous-in-speed state, and the totally heterogeneous state [11]. The homogeneous-in-speed state especially demonstrates the fact that, in contrast to the free flow, there is no direct relationship between the density and flux of vehicles in the synchronized mode because their variations are totally uncorrelated [11]. For example, an increase in the vehicle density can be accompanied by either an increase or decrease in the vehicle flux, with the car velocity being practically constant. As a result, the synchronized mode corresponds to a two-dimensional region in the flow-density plane ($j\rho$ plane) rather than to a certain line $j=\vartheta(\rho)\rho$ [11]. Keeping in mind a hypothesis by Kerner [8,9,34] about the metastability of each particular state in this synchronized mode region it is natural to assume that there should be at least one additional state variable affecting the vehicle flux. The other important feature of the synchronized mode is the key role of some cars bunched together and traveling much faster than the typical ones, which enables us to regard them as a special car group [11]. Therefore, in the synchronized mode the function of car distribution in the velocity space should have two maxima and we will call such fast car groups platoons in speed.

Anomalous properties of the synchronized mode have been substantiated also in [33] using single-car data. In particular, as the car density comes to the critical value ρ_c of the free flow ↔ synchronized mode transition the time-headway distribution exhibits a short-time peak (at 0.8 s). This short-time headway corresponds to '' . . . platoons of some vehicles traveling very fast—their drivers are taking the risk of driving ''bumper-to-bumper'' with a rather high speed. These platoons are the reason for the occurrence of high-flow states in free traffic'' [33]. The platoons are metastable and their destruction gives rise to the congested vehicle motion [35]. In the synchronized mode the weight of the short-time headways is less; however, almost every fourth driver falls below the 1 s threshold. In the vicinity of the free flow $\leftrightarrow$synchronized mode transition the short time-headways have the greatest weight. In other words, at least near the given phase transition the traffic flow state is to be characterized by two different driver groups which separate from each other in the *velocity* space and, consequently, in multilane traffic flow there should be another relaxation process distinct from one taken into account by the model (1),(2). In order to move faster than the statistically averaged car a driver should permanently maneuver pass by the cars moving ahead. The meeting of several such ''fast'' drivers seems to cause the platoon formation. Obviously, to drive in such a manner requires additional efforts, so, each driver needs a certain time τ to get the decision whether or not to take part in these maneuvers. Exactly the time τ characterizes the relaxation processes in the platoon evolution. It should be noted that the overtaking maneuvers are not caused by the control over the headway distance and, thus, the corresponding transient processes may be much slower than the driver response to variations in the headway to prevent possible traffic accidents.

The analysis of the obtained optimal-velocity function $V(\Delta x)$ demonstrates its dependence not only on the headway Δx but also on the local car density. So, in congested flow the drivers supervise the vehicle arrangement or, at least, try to do this in a sufficiently large neighborhood covering several lanes.

Another unexpected fact is that the synchronized mode is mainly distinctive not due to the car velocities at different lanes being equal. In the observed traffic flow various lanes did not exhibit a substantial difference in the car velocity even in the free flow. In agreement with the results obtained by Kerner [11] the synchronized mode is singled out by small correlations between fluctuations in the car flow, velocity, and density. There is only a strong correlation between the velocities at different lanes taken at the same time; however, it decreases sufficiently fast as the time difference increases. By contrast, there are strong long-time correlations between the flow and density for the free flow as well as the stop-and-go mode. In these phases the vehicle flow directly depends on the density.

Thereby, the free flow, the synchronized mode, and the jammed motion seem to be qualitatively distinct from one another at the microscopic level. So, it is likely that to describe macroscopically traffic phase transitions the set of the state variables $\{\rho,v,\theta\}_\alpha$ should be completed with an additional parameter (or parameters) reflecting the *internal* correlations in the car dynamics. In other words, this parameter has to be due to the ''many-body'' effects in the car interaction in contrast to such *external* variables as the mean car density and velocity being actually the zeroth and first moments of the ''one-particle'' distribution function. Thus, it can be regarded as an independent state variable of traffic flow. The derivation of macroscopic traffic equations based on a Boltzmann-like kinetic approach [36] has also shown that there is an additional internal degrees of freedom in the vehicle dynamics.

In any case a theory of unstable traffic flow has to answer, in particular, to a question of why its two phases, e.g., the free flow and the synchronized mode, can coexist and, thus, what is the difference between them as well as why the separating transition region (Fig. 1) does not widen but keeps a certain thickness. Besides, it should specify the velocity u of this region depending on the traffic phase characteristics. There is a general expression relating the transition region velocity u to the density and mean velocity of cars in the free flow and a developed car cluster: ρ_f, v_f and ρ_{cl}, v_{cl}, respectively, that follows from the vehicle conservation [1], namely, the Lighthill-Whitham formula

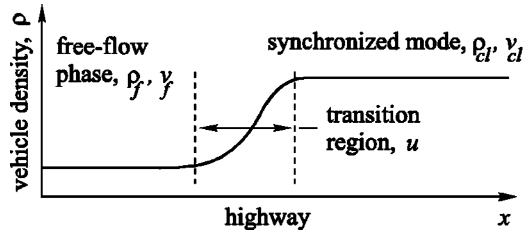

FIG. 1. Transition region separating, e.g., the free-flow and synchronized mode.

$$u=\frac{\rho_{cl}v_{cl}-\rho_f v_f}{\rho_{cl}-\rho_f}. \tag{10}$$

A specific model is to give additional relationships between the quantities u, ρ_f, v_f and ρ_{cl}, v_{cl} resulting from particular details of the car interaction. We note that a description similar to Eqs. (1) and (2) dealing solely with the external parameters $\{\rho,v\}$ do not actually make a distinction between the free-flow and congested phases and their coexistence is due to the particular details of the car interaction.

The free flow $\leftrightarrow$ synchronized motion transition is rather similar to aggregation phenomena in physical systems such as undercooled liquid when in a metastable phase (undercooled liquid) the transition to a new ordered (crystalline) phase goes through the formation of small clusters. Keeping in mind this analogy Mahnke and co-workers [37–39] have proposed a kinetic approach based on a stochastic master equation describing the synchronized mode formation that deals with individual free cars and their clusters. The cluster evolution is governed by the attachment and evaporation of the individual cars and the synchronized mode is regarded as the motion of a large cluster.

To describe such phenomena in physical systems an effective macroscopic approach was developed, called the Landau phase transition theory [40], that introduces a certain order parameter h characterizing the correlations, e.g., in the atom arrangement. In the present paper following practically the spirit of the Landau theory we develop a phenomenological approach to the description of the traffic flow instability that ascribes to the vehicle flux an additional *internal* parameter which will be also called the order parameter h and which allows for the effect of lane changing on the vehicle motion. In this way the free flow and the congested phases become in fact distinctive and solely the conditions of their coexistence and the dynamics of the transition layer are the subject of specific models.

II. ORDER PARAMETER AND THE INDIVIDUAL DRIVER BEHAVIOR

We describe the vehicle flow on a multilane highway in terms of its characteristics averaged over the road cross sec-

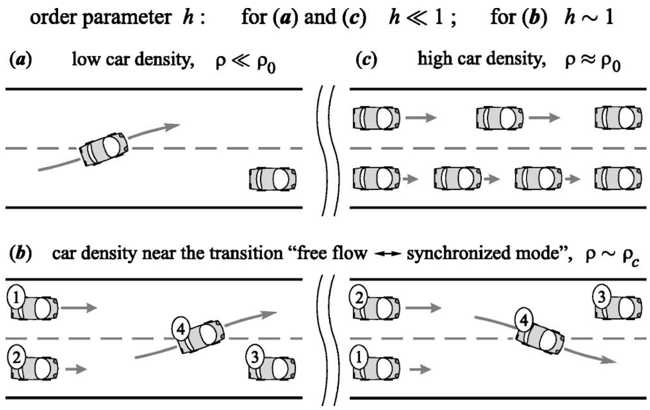

FIG. 2. Schematic illustration of the car arrangement in the various phases of traffic flow and the multilane vehicle interaction caused by car overtaking.

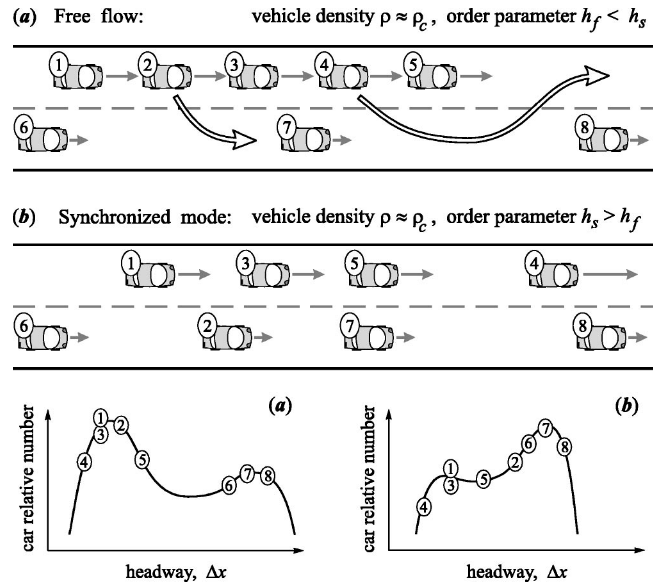

FIG. 3. Schematic illustration of the alteration in the vehicle arrangement near the free flow ↔ synchronized mode phase transition.

tion, namely, by the car density ρ, the mean velocity v, and the order parameter h. The latter is the measure of the correlations in the car motion or, what is equivalent, of the car arrangement regularity forming due to the lane change by the ''fast'' drivers. Let us discuss the physical meaning of the order parameter h in detail considering individually the free flow, synchronized mode, and jammed traffic (Fig. 2).

A. Physical meaning of the order parameter *h* and its governing equation

When vehicles move on a multilane highway without changing the lanes they interact practically with the nearest neighbors ahead only and, so, there should be no internal correlations in the vehicle flow at different lanes. Therefore, although under this condition the traffic flow can exhibit complex behavior, for example, the ''stop-and-go'' waves can develop, it is actually of a one-dimensional nature. In particular, the drivers that would prefer to move faster than the statistically mean driver will bunch up forming the platoons headed by a relatively slower vehicle. When the cars begin to change lanes for overtaking slow vehicles the car ensembles at different lanes will affect one another. The case of this interaction is due to that a car during a lane change maneuver occupies, in a certain sense, two lanes simultaneously, affecting the cars moving behind it in both the lanes. Figure 2(b) illustrates this interaction for cars 1 and 2 through car 4 changing the lanes. The drivers of both cars 1 and 2 have to regard car 4 as the nearest neighbor and, so, their motion will be correlated during the given maneuver and after it during the relaxation time τ'. In the same way car 1 is affected by car 3 because the motion of car 4 directly depends on the behavior of car 3. The more frequently lane changing is performed, the more correlated traffic flow there is on a multilane highway. Therefore, it is reasonable to introduce the order parameter h being the mean density of such car triplets normalized to its maximum possible for the given highway and to regard it as a measure of the multilane correlations in the vehicle flow.

On the other hand, the order parameter h introduced in this way can be regarded as a measure of the vehicle arrangement regularity. Let us consider this question in detail for the free flow, synchronized mode, and jammed traffic individually. In the free flow the feasibility of overtaking makes the vehicle arrangement more regular because of platoon dissipation. So as the order parameter h grows the free traffic becomes more regular. Nevertheless, in this case the density of the car mulitlane triplets remains relatively low, $h \ll 1$, and the vehicle ensembles should exhibit weak correlations. Whence it follows also that the mean car velocity ϑ is an increasing function of the order parameter h in the free flow. In the jammed motion [Fig. 2(c)] leaving current lanes is hampered because of lack of room for the maneuvers. So the car ensembles at different lanes can be mutually independent in spite of individual complex behavior. In the given case the order parameter must be small, too, $h \ll 1$, but, in contrast, the car mean velocity should be a decreasing function of h. In fact, for highly dense traffic any lane change of a car requires practically that the neighbor drivers decelerate giving a place for this maneuver.

Figure 3 illustrates the free flow ↔ synchronized mode transition. As the car density grows in free flow, the fast drivers that at first overtake slow vehicles individually begin to gather into platoons headed by more slow cars among them but, nevertheless, moving faster than the statistically mean vehicle [Fig. 3(a)]. The platoons are formed by drivers preferring to move as fast as possible keeping short headways without lane changing. Such a state of the traffic flow should be sufficiently inhomogeneous and the vehicle headway distribution has to contain a short headway spike as observed experimentally in [33]. Therefore, even at a sufficiently high car density the free flow should be characterized by weak multilane correlations and not too great values of the order parameter h_f. The structure of these platoons is also inhomogeneous; they comprise cars whose drivers would prefer to move at different headways (for a fixed velocity) under comfortable conditions, i.e., when the cars moving behind a given car do not jam it or none of the vehicles moving on the neighboring lanes hinders its motion at the given velocity provided it changes the current lane. So, when the density of vehicles attains sufficiently high values and their mean velocity decreases remarkably with respect to the velocity on the empty highway some of the fast drivers can decide that there is no reason to move so slowly at such short headways requiring strain. Then they can either overtake the car heading the current platoon by changing lanes individually or leave the platoon and take vacant places [Fig. 3(a)]. The former has to increase the multilane correlations and, in part, to decrease the mean vehicle velocity because the other drivers should give place for this maneuvers in a sufficiently dense traffic flow. The latter also will decrease the mean vehicle velocity because these places were vacant from the standpoint of sufficiently fast drivers only but not from the point of view of the statistically mean ones, preferring to keep longer headways in comparison with the platoon headways. Therefore, the statistically mean drivers have to decelerate, decreasing the mean vehicle velocity. The two maneuver types make the traffic flow more homogeneous dissipating the platoons and smoothing the headway distribution [Fig. 3(b) and the low fragment]. Besides, the single-vehicle experimental data [33] show that the synchronized mode is singled out by long-distant correlations in the vehicle velocities, whereas the headway fluctuations are correlated only on small scales, which justifies the assumptions of the synchronized mode being a more homogeneous state than the free flow. We think that the given scenario describes the synchronized mode formation which must be characterized by a great value of the order parameter, $h_s > h_f$, and a lower velocity in comparison with the free flow at the same vehicle density.

In addition, whence it follows that first the left boundary of the headway distribution should be approximately the same for both the free flow and the synchronized mode near the phase transition, which corresponds to the experimental data [33]. Second, since in this case the transition from the free flow to the synchronized mode leads to the decrease in the mean velocity, the fast driver will see no reason to alter their behavior and to move forming platoons again until the vehicle density decreases and the mean velocity grows enough. It is reasonable to relate this characteristics to the experimentally observed hysteresis in the free flow $\leftrightarrow$ synchronized mode transition [12–14]. Third, for a car to be able to leave a given platoon the local vehicle arrangement at the neighboring lane should be of special form and when an event of the vehicle rearrangement occurs its following evolution depends also on the particular details of the neighboring car configuration exhibiting substantial fluctuations. Therefore, the synchronized mode can comprise a great amount of local metastable states and corresponds to a certain two-dimensional region on the flow-density plane ($j\rho$ plane) rather than a line $j = \vartheta(\rho)\rho$, which matches the experimental data [11] and the modern notion of the synchronized mode nature [7–9]. This feature seems to be similar to that met in physical media with local order, for example, in glasses where phase transitions are characterized by a wide range of controling parameters (temperature, pressure, etc.) rather than their fixed values (see, e.g., [41]).

This uncertainty of the synchronized mode, at least qualitatively, may be regarded as an effect of the internal fluctuations of the order parameter h and at the first step we will ignore them assuming the order parameter h to be determined in a unique fashion for fixed values of the vehicle density ρ and the mean velocity v. Thus for a uniform vehicle flow we write

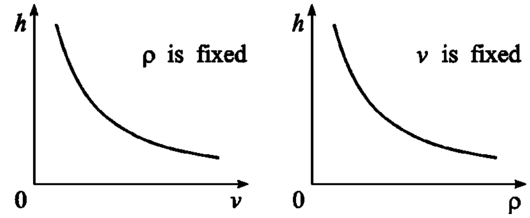

FIG. 4. Qualitative sketches of the order parameter h as a function of the vehicle mean velocity v and the density ρ specified by the behavior of individual drivers.

$$\tau \frac{dh}{dt} = -\Phi(h,\rho,v), \tag{11}$$

where τ is the time required of drivers coming to the decision to begin or stop overtaking maneuvers and the function $\Phi(h,\rho,v)$ possesses a single stationary point $h = h(\rho,v)$ being stable and, thus,

$$\frac{\partial \Phi}{\partial h} > 0. \tag{12}$$

The latter inequality is assumed to hold for all the values of the order parameter for simplicity. We note that Eq. (11) also allows for the delay in the driver response to changes on the road. However, in contrast with models similar to Eqs. (1) and (2), here this effect is not the origin of the traffic flow instability and, thus, its particular description is not so crucial. Moreover, as discussed in the Introduction, the time τ characterizes the delay in the driver decision concerning the lane changing but not the control over the headway, enabling us to assume $\tau \gg \tau'$.

The particular value $h(v,\rho)$ of the order parameter results from the compromise between the danger of the accident during changing lanes and the will to move as fast as possible. Obviously, the lower the mean vehicle velocity v is for a fixed value of ρ, the weaker is the lane changing danger and the stronger is the will to move faster. Besides, the higher the vehicle density ρ is for a fixed value of v, the stronger is this danger (here the will has no effect at all). These statements enable us to regard the dependence $h(v,\rho)$ as a decreasing function of both the variables v, ρ (Fig. 4) and we take into account the inequality (12) to write

$$\frac{\partial \Phi}{\partial v} > 0, \quad \frac{\partial \Phi}{\partial \rho} > 0, \tag{13}$$

with the latter inequality stemming from the danger effect only.

Equation (11) describes actually the behavior of the drivers who prefer to move faster than the statistically mean vehicle and whose readiness for risk is greatest. Exactly this group of drivers governs the value of h. There is, however, another characteristic of the driver behavior; it is the mean velocity $v = \vartheta(h,\rho)$ chosen by the *statistically averaged* driver also taking into account the danger resulting from the frequent lane changing by the fast drivers. This characteristic is actually the same as the one discussed in the Introduction

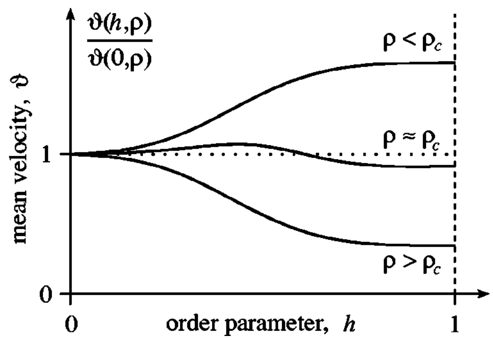

FIG. 5. A qualitative sketch of the mean vehicle velocity vs the order parameter h for several fixed values of the vehicle density ρ.

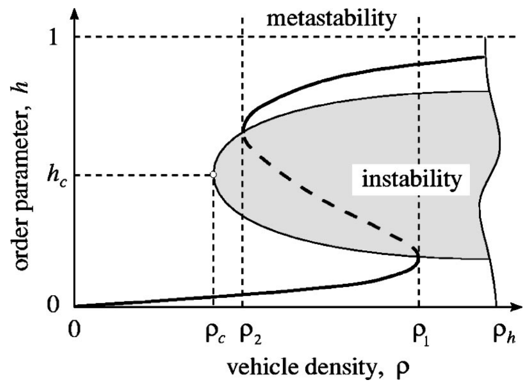

FIG. 6. The region of the traffic flow instability in the $h\rho$ plane and the form of the curve $h(\rho)$ displaying the dependence of the order parameter on the vehicle density. The plot is a qualitative sketch.

but also depends on the order parameter h. So, as a function of ρ it meets conditions (4). Concerning the dependence of $\vartheta(h,\rho)$ on h we can state that generally this function should be increasing for small values of the car density, $\rho \ll \rho_0$, because in the given case the lane changing practically makes no danger to traffic and all the drivers can overtake vehicles moving at lower speed without risk. By contrast, when the vehicle density is sufficiently high, $\rho \lesssim \rho_0$, only the most ''impatient'' drivers permanently change the lanes for overtaking, making an additional danger to the most part of the other drivers. Therefore, in this case the velocity $\vartheta(h,\rho)$ has to decrease as the order parameter h increases. For certain intermediate values of the vehicle density, $\rho \approx \rho_c$, this dependence is to be weak. Figure 5 shows the velocity $\vartheta(h,\rho)$ as a function of h for different values of ρ, where, in addition, we assume the effect of the order parameter $h \in (0,1)$ near the boundary points is weak and we set

$$\frac{\partial \vartheta}{\partial h} = 0 \quad \text{at} \quad h=0 \text{ and } h=1. \tag{14}$$

We will ignore the delay in the relaxation of the mean velocity to the equilibrium value $v = \vartheta(h,\rho)$ because the corresponding delay time characterizes the driver control over the headway and should be short, as already discussed above. Then the governing equation (11) for the order parameter h can be rewritten in the form

$$\tau \frac{dh}{dt} = -\phi(h,\rho); \quad \phi(h,\rho) \overset{\text{def}}{=} \Phi[h,\rho,\vartheta(h,\rho)]. \tag{15}$$

For the steady-state uniform vehicle flow the solution of the equation $\phi(h,\rho)=0$ specifies the dependence $h(\rho)$ of the order parameter on the car density. Let us now study its properties and stability.

B. Nonmonotony of the $h(\rho)$ dependence and the traffic flow instability

To study the local characteristics of the right-hand side of Eq. (15) we analyze its partial derivatives

$$\frac{\partial \phi}{\partial h} = \frac{\partial \Phi}{\partial h} + \frac{\partial \Phi}{\partial v}\frac{\partial \vartheta}{\partial h}, \tag{16}$$

$$\frac{\partial \phi}{\partial \rho} = \frac{\partial \Phi}{\partial \rho} + \frac{\partial \Phi}{\partial v}\frac{\partial \vartheta}{\partial \rho}. \tag{17}$$

As mentioned above, the value of $\partial\Phi/\partial\rho$ is solely due to the danger during changing lanes, so this term can be ignored until the vehicle density ρ becomes sufficiently high. In other words, in a certain region $\rho < \rho_h \lesssim \rho_0$ the derivative $\partial\phi/\partial\rho \sim (\partial\Phi/\partial v)(\partial\vartheta/\partial\rho) < 0$ by virtue of Eqs. (4) and (13). So, the local behavior of the function $h(\rho)$ [meeting the equality $d\phi = 0$ and, thus, $dh/d\rho = -(\partial\phi/\partial\rho)(\partial\phi/\partial h)^{-1}$] depends directly on the sign of the derivative $\partial\phi/\partial h$; it is increasing or decreasing for $\partial\phi/\partial h > 0$ or $\partial\phi/\partial h < 0$, respectively.

For long-wave perturbations proportional to $\exp\{ikx\}$ of the car distribution on a highway, the density ρ can be treated as a constant at the lower order in k. Therefore, according to Eq. (15) the steady-state traffic flow is unstable if $\partial\phi/\partial h < 0$.

Due to Eqs. (12) and (14) the first term on the right-hand side of Eq. (16) is dominant in the vicinity of the lines $h=0$ and $h=1$, thus in this region the curve $h(\rho)$ is increasing and the stationary state of the traffic flow is stable. For $\rho < \rho_c$ the value $\partial\vartheta/\partial h > 0$ (Fig. 5); therefore, the whole region $\{0 < h < 1, 0 < \rho < \rho_c\}$ corresponds to the stable car motion. However, for $\rho > \rho_c$ there can be a region of the order parameter h where the derivative $\partial\phi/\partial h$ changes the sign and the vehicle motion becomes unstable. Indeed, the solution $v = \eta(h,\rho)$ of the equation $\Phi(h,\rho,v) = 0$ can be regarded as the mean vehicle velocity controlled by the fast drivers and is a decreasing function of h because of $\partial\eta/\partial h = -(\partial\Phi/\partial h)/(\partial\Phi/\partial v)^{-1}$. So, once such ''active'' drivers start to change lanes to move faster, they will do this as frequently as possible especially if the mean velocity decreases, which corresponds to a considerable increase in h for a small decrease in v. So, it is quite natural to assume that the value of $\partial\eta/\partial h$ for $\rho > \rho_c$ is sufficiently small and

$$\frac{\partial \phi}{\partial h} = \frac{\partial \Phi}{\partial v}\left(\frac{\partial \vartheta}{\partial h} - \frac{\partial \eta}{\partial h}\right) < 0. \tag{18}$$

Under these conditions the instability region does exist, the curve $h(\rho)$ can look like S (Fig. 6), and its decreasing branch corresponds to the unstable vehicle flow. The lower increasing branch matches the free flow state of the car motion, whereas the upper one should be related to the synchronized mode because it is characterized by the order parameter coming to unity.

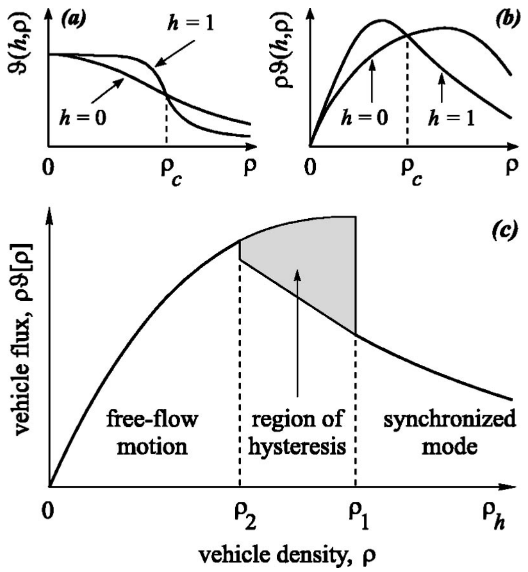

FIG. 7. The mean vehicle velocity (a) and the vehicle flux (b) vs the vehicle density for the limit values of the order parameter $h=0$ and $h=1$ as well as the resulting fundamental diagram (c). The plot is a qualitative sketch.

C. Hysteresis and the fundamental diagram

The obtained dependence $h(\rho)$ actually describes the first-order phase transition in the vehicle motion. Indeed, when increasing the car density exceeds the value ρ_1 the free flow becomes absolutely unstable and the synchronized mode forms through a sharp jump of the order parameter. If, however, after that the car density decreases the synchronized mode will persist until the car density attains the value $\rho_2<\rho_1$. It is a typical hysteresis and the region (ρ_2,ρ_1) corresponds to the metastable phases of traffic flow.

Let us now discuss a possible form of the fundamental diagram $j=j(\rho)$ showing the vehicle flux $j=\rho\vartheta[\rho]$ as a function of the car density ρ, where, by definition, $\vartheta[\rho]=\vartheta[h(\rho),\rho]$. It should be pointed out that here we confine our consideration to the region of not too large values of the car density, $\rho<\rho_h$, where the free flow $\leftrightarrow$ synchronized mode transition takes place. The synchronized mode $\leftrightarrow$jammed traffic transition will be discussed below. Figure 7(a) displays the dependence $\vartheta(h,\rho)$ of the mean vehicle velocity on the density ρ for the fixed limit values of the order parameter $h=0$ and 1. For small values of ρ these curves practically coincide with each other. As the vehicle density ρ grows and until it comes close to the critical value ρ_c when the lane change danger becomes substantial, the velocity $\vartheta(1,\rho)$ practically does not depend on ρ. So at the point ρ_c at which the curves $\vartheta(1,\rho)$ and $\vartheta(0,\rho)$ meet each other, $\vartheta(1,\rho)$ is to exhibit a sufficiently sharp decrease in comparison with the latter one. Therefore, on one hand, the function $j_1(\rho)=\rho\vartheta(1,\rho)$ has to be decreasing for $\rho>\rho_c$. On the other hand, at the point ρ_c for $h\ll 1$ the effect of the lane change danger is not extremely strong; it only makes the lane change ineffective, $\partial\vartheta/\partial h\approx 0$ (Fig. 5). So, it is reasonable to assume the function $j_0(\rho)=\rho\vartheta(0,\rho)$ increases near the point ρ_c. Under the adopted assumptions the relative arrangement of the curves $j_0(\rho)$, $j_1(\rho)$ is demonstrated in Fig. 7(b), and Fig. 7(c) shows the fundamental diagram of traffic flow resulting from Figs. 6 and 7(b).

Concluding the present section we note that in the given description of the driver behavior governing the order parameter h the vehicle flux $j(h,\rho)=\rho\vartheta(h,\rho)$ is an external characteristic of traffic flow. So, the obtained form of the fundamental diagram does not follow directly from the developed model, but can be interpreted sufficiently reasonable. It can be rigorously justified if the critical point ρ_c corresponds to the maximum of the flux $j(h^*,\rho)$ for a certain fixed value h^* of the order parameter. In other words, when the road capacity is exhausted and the following increase in the vehicle density leads to a decrease in the vehicle flux the drivers divide into two groups; the majority prefer to move at their own lanes whereas the most ‘‘impatient’’ drivers change the lanes as frequently as possible, giving rise to the traffic instability. This problem, however, deserves an individual investigation.

III. PHASE COEXISTENCE. DIFFUSION-LIMITED CLUSTER MOTION

The preceding section has considered uniform traffic flow, so we analyzed actually the individual characteristics of the free flow and the synchronized mode. In the present section we study their coexistence, i.e., the conditions under which a car cluster of finite size forms. This problem, however, requires that the traffic flow model be defined concretely. Therefore, in what follows we will consider a certain simple model which illustrates the characteristic features of the car cluster self-organization without complex mathematical manipulations.

As before, the model under consideration assumes the mean velocity relaxation to be immediate and modifies the governing equation (15) in such a way as to ascribe the order parameter h to a local car group. In other words, we describe the vehicle flow by the Lighthill-Whitham equation with dissipation (see, e.g., [42] and also the Introduction), we replace the time derivative in Eq. (15) by the particle derivative, and we take into account that the order parameter cannot exhibit substantial variations over scales $l\sim\theta^{1/2}\tau\lesssim v_0\tau$ (θ is the velocity variance, v_0 is the typical car velocity in the free flow). Namely, we write

$$\frac{\partial\rho}{\partial t}+\frac{\partial[\rho\vartheta(h,\rho)]}{\partial x}=D\frac{\partial^2\rho}{\partial x^2}, \tag{19}$$

$$\tau\left[\frac{\partial h}{\partial t}+\vartheta(h,\rho)\frac{\partial h}{\partial x}\right]=\hat{\mathcal{L}}\{h\}-\phi(h,\rho)+\xi(x,t). \tag{20}$$

Let us discuss the meaning of the particular terms of the given model. The Burgers equation (19), as already discussed in Introduction, allows for the fact that drivers govern their motion taking into account not only the behavior of the nearest cars, but the state of traffic flow inside the whole field of their front view of length. The effective diffusivity D can be estimated as $D\sim L^*v_0$, where $L^*\gg l$ is a front distance looked through by drivers assumed to be much greater than the scale l, so

$$D\tau\sim lL^*\gg l^2. \tag{21}$$

The function $\phi(h,\rho)$ is of the form

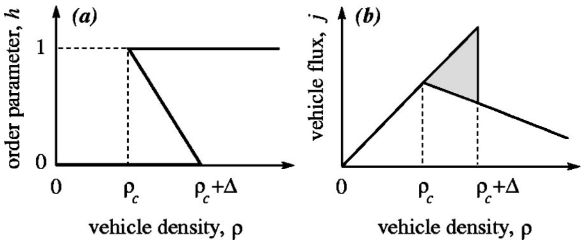

FIG. 8. The dependence $h(\rho)$ and the fundamental diagram of traffic flow described by the model (19),(20).

$$\phi(h,\rho)\overset{\mathrm{def}}{=}h(1-h)[a(\rho)-h], \tag{22}$$

where

$$a(\rho)=\begin{cases}1 & \text{for } \rho<\rho_c\\ (\rho_c+\Delta-\rho)/\Delta & \text{for } \rho_c<\rho<\rho_c+\Delta\\ 0 & \text{for } \rho>\rho_c+\Delta.\end{cases}$$

It describes such a driver behavior that $h=0$ and $h=1$ are the unique stable values of the order parameter for $\rho<\rho_c$ and $\rho>\rho_c+\Delta$, respectively, whereas for $\rho_c<\rho<\rho_c+\Delta$ the points $h=0$, $h=1$ are both locally stable and there is an additional unstable stationary point, namely, $h=a(\rho)$. The term

$$\hat{\mathcal{L}}\{h\}\overset{\mathrm{def}}{=}l^2\frac{\partial^2 h}{\partial x^2}+\frac{l}{\sqrt{2}}\frac{\partial h}{\partial x} \tag{23}$$

governs spatial variations in the field $h(x,t)$ and takes into account that drivers mainly follow the behavior of cars in front of them and cars moving at the rear cannot essentially affect them. The mean car velocity depends on h and ρ as

$$\rho\vartheta(h,\rho)\overset{\mathrm{def}}{=}\rho\vartheta_0(1-h)+[\rho_c\vartheta_0-\nu(\rho-\rho_c)]h. \tag{24}$$

The last term on the right-hand side of Eq. (20) characterizes the random fluctuations in the order parameter dynamics,

$$\langle\xi(x,t)\rangle=0, \tag{25}$$

$$\langle\xi(x,t)\xi(x',t')\rangle=\sigma^2 l\tau\delta(x-x')\delta(t-t'), \tag{26}$$

where σ is their dimensionless amplitude. Expressions (22) and (24) give the $h(\rho)$ dependence and the fundamental diagram shown in Fig. 8 simplifies the one presented in Fig 7.

If we ignore the random fluctuations of the order parameter h, i.e., set $\sigma=0$, then the model (19),(20) will give us an artificially long delay (much greater than τ) in the order parameter variations from, for example, the unstable point $h=0$ to the stable point $h=1$. Such a delay can lead to a meaningless great increase of the vehicle density in the free flow without phase transition to congestion. In order to avoid this artifact and to allow for the effect of real fluctuations in the driver behavior we also will assume the amplitude σ to obey the condition [43]:

$$\left(\frac{l}{L^*}\right)^{5/4}\lesssim\sigma\ll 1 \tag{27}$$

($\sigma\ll 1$, because, otherwise, the traffic flow dynamics would be totally random). It should be noted that small random variations of the order parameter h near the points $h=0$, $h=1$ going into the regions $h<0$ and $h>1$, respectively, do not come into conflict with its physical meaning as the measure of the car motion correlations. Indeed, the chosen values $h=0$ and $h=1$ can describe a renormalization of real correlation coefficients $\tilde{h}=\tilde{h}_1>0$, $\tilde{h}_2<1$.

According to Eq. (20), for the order parameter h the characteristic scale of its spatial variations is l, so, the layer $\Im_h$ separates the regions where $h\approx 0$ and 1 is of thickness about l. Due to inequality (21) the car density on such scales can be treated as constant. Therefore, the transition region $\mathcal{L}_\rho$ between practically the uniform free flow and the congested phase is of thickness determined mainly by spatial variations of the vehicle density and on such scales the layer $\Im_h$ can be treated as an infinitely thin interface. In addition, the characteristic time scale of the layer $\Im_h$ formation is about τ, whereas it takes about the time $\tau_\rho\sim D/v_0^2\sim\tau(L^*/l)\gg\tau$ for the layer $\mathcal{L}_\rho$ to form. Thereby, when analyzing the motion of wide car clusters we may regard the order parameter distribution $h(x,t)$ as quasistationary for a fixed value of the car density ρ. Let us now consider two possible limits of the layer $\Im_h$ motion under such conditions.

A. Regular dynamics

In the region $\rho_c<\rho<\rho_c+\Delta$ until the value of $a(\rho)$ comes close to the boundaries $h=0$ and $h=1$ the effect of the random fluctuations is ignorable. In this case by virtue of the adopted assumptions the solution of Eq. (20) that describes the layer $\Im_h$ moving at the speed u is of the form

$$h=\frac{1}{2}\left[1+\tanh\left(\frac{x-ut}{\lambda}\right)\right]. \tag{28}$$

Here for the layer $\Im_{01}$ of the free-flow → synchronized mode transition and for the layer $\Im_{10}$ of the opposite transition (Fig. 9)

$$\lambda_{01}=\frac{2\sqrt{2}}{\eta_v}l, \quad \lambda_{10}=-2\sqrt{2}\eta_v l, \tag{29}$$

$$u_{01}=\vartheta_0-\frac{\Delta_v}{2}-\frac{l}{\sqrt{2}\eta_v\tau}[1+\eta_v-2a(\rho_i)], \tag{30}$$

$$u_{10}=\vartheta_0-\frac{\Delta_v}{2}-\frac{l}{\sqrt{2}\tau}[2\eta_v a(\rho_i)-(\eta_v-1)], \tag{31}$$

where we introduced the quantities

$$\Delta_v=\vartheta(0,\rho_i)-\vartheta(1,\rho_i),$$

$$\eta_v=\left[1+\left(\frac{\tau\Delta_v}{2\sqrt{2l}}\right)^2\right]^{1/2}+\frac{\tau\Delta_v}{2\sqrt{2l}},$$

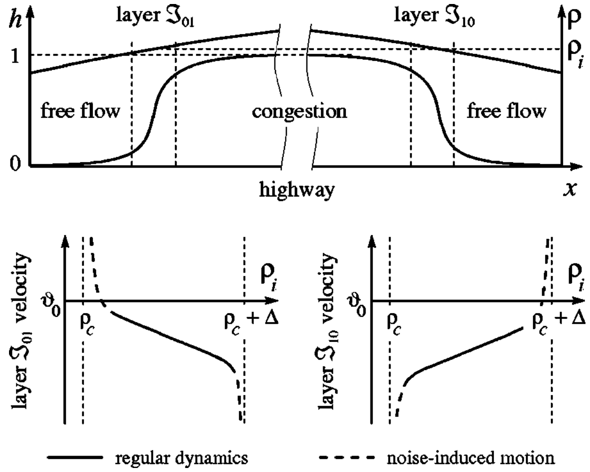

FIG. 9. The distribution of the order parameter and the car density in the vicinity of the layers $\Im_h$ of the transition between the free flow and the synchronized phase as well as the velocity of their motion vs the local values ρ_i of the car density.

and ρ_i is the corresponding value of the car density inside the layers $\Im_{01}$ and $\Im_{10}$.

Expressions (30) and (31) describe the regular dynamics of the car cluster formation because the transition, for example, from the free flow to the synchronized phase at a certain point x is induced by this transition at the nearest points. The dependence of the velocities u_{01} and u_{10} on the local car density ρ_i is illustrated in Fig. 9. The characteristic velocities attained in this type of motion can be estimated as

$$\vartheta_0 - u \sim \max\{\vartheta_0 \Delta/\rho_c, \, l/\tau\},$$

so, under the adopted assumptions the regular dynamics does not allow for the sufficiently fast motion of the layers $\Im_h$ upstream.

B. Noise-induced dynamics

As the car density ρ tends to the critical values ρ_c or $\rho_c+\Delta$ the value of $a(\rho)$ comes close to the boundaries $a(\rho_c)=1$ and $a(\rho_c+\Delta)=0$, and the point $h=1$ or $h=0$ becomes unstable, respectively. In this case the effect of the random fluctuations $\xi(x,t)$ plays a substantial role. Namely, the phase transition, for example, from the free flow to the synchronized motion (for $\rho \approx \rho_c+\Delta$) is caused by the noise $\xi(x,t)$ and equiprobably takes place at every point of the region wherein $\rho \approx \rho_c+\Delta$ rather than is localized near the current position of the layer $\Im_{01}$. Under these conditions the motion of the layers $\Im_h$ can be qualitatively characterized by an extremely high velocity in both the directions, which is illustrated in Fig. 9 by dashed lines.

We note that the noise-induced motion, in contrast to the regular dynamics, is to exhibit significant fluctuations in the displacement of the layer $\Im_h$ as well as in its forms. This question is, however, a subject for individual study.

C. Diffusion-limited motion of vehicle clusters

Let us now analyze the motion of a sufficiently large cluster that can form on a highway when the initial car density or, what is the same, the average car density $\bar{\rho}$ belongs to the metastable region, $\bar{\rho} \in (\rho_c, \rho_c+\Delta)$. The term ''sufficiently large'' means that the cluster dimension L is assumed to be much greater than the front distance L^* looked through by drivers so they cannot look round the congestion as a whole. In this case a quasilocal description of traffic flow similar to the differential equations (19) and (20) is justified.

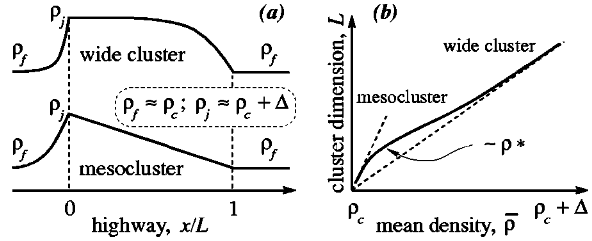

FIG. 10. The possible forms of the car clusters and their dimension vs the mean car density.

Converting to the frame $y = x - ut$ moving at the cluster velocity u, solving Eq. (19) individually for the free flow and the synchronized phase, and treating the layers $\Im_h$ as infinitely thin interfaces we get the following conclusion. Within the framework of the given model the car cluster moves upstream sufficiently fast, so the motion of the layers $\Im_{01}$ and $\Im_{10}$ is governed by the noise $\xi(x,t)$. In this case the values of the car density at the layers $\Im_{01}$ and $\Im_{10}$ have to be $\rho_j \approx \rho_c+\Delta$ and $\rho_f \approx \rho_c$, respectively. Thereby, the cluster velocity u is mainly determined by the car redistribution governed by the diffusion-type processes. The latter feature is the reason why we refer to the cluster dynamics under such conditions as to the diffusion-limited motion. The transition region $\mathcal{L}_{01}$ between practically the uniform free-flow state and the cluster contains the exponential increase of the vehicle density inside the free-flow phase from the value ρ_f far from the ''interface'' $\Im_{01}$ up to $\rho_j \approx \rho_c+\Delta$ at $\Im_{01}$,

$$\rho = \rho_f + (\rho_j - \rho_f)\exp\{q_f y\},$$

where $q_f = (\vartheta_0 + |u|)/D \sim 1/L^*$ and the frame $\{y\}$ is attached to the interface $\Im_{01}$. The transition region $\mathcal{L}_{10}$ from the synchronized phase to the uniform free flow is to be localized inside the car cluster. So, it is characterized by the decrease in the vehicle density $\delta\rho \propto \exp\{q_j y\}$, where $q_j = (|u| - \nu)/D$, and the vehicle free flow leaving the cluster is uniform at all its points [Fig. 10(a)].

The cluster velocity is directly determined by the motion of the interface $\Im_{01}$. Therefore, assuming also the cluster dimension L large in comparison with L^*, from Eq. (19) we get the expression of the same form as the Lighthill-Whitham formula (10) relating the cluster velocity u and the vehicle flux characteristics on both sides of the layer $\mathcal{L}_{01}$. Whence it follows that at the first approximation

$$u \approx -\nu, \tag{32}$$

the value $q_j = 0$, and the vehicle cluster is of the form shown in Fig. 10(a) under the name ''mesocluster.'' Assuming the total number of cars on the highway of length L_{rd} fixed we get the expression for the mesocluster dimension L,

$$L = 2L_{\mathrm{rd}} \frac{\bar{\rho} - \rho_c}{\Delta}. \tag{33}$$

However, this result is justified only for sufficiently small values of $(\overline{\rho}-\rho_c)/\Delta \ll 1$, when the cluster dimension is not too large, $Lq_j \ll 1$ (nevertheless, $L \gg L^*$). Exactly for this reason we refer to such clusters as mesoscopic ones. In order to study the opposite limit, $Lq_j \gg 1$, we have to take into account that the value ρ_f is not rigorously equal to ρ_c but practically is the root $\rho_f^* > \rho_c$ of the equation $u_{10}(\rho_f^*) = -\nu$. In this case the Lighthill-Whitham formula (10) gives the expression

$$u \simeq -\left[\nu + (\vartheta_0 + \nu)\frac{\rho_f^* - \rho_c}{\Delta}\right],$$

leading to the following estimates of the thickness $1/q_j$ of the transition region $\mathcal{L}_{10}$:

$$1/q_j \sim \frac{D\Delta}{(\vartheta_0+\nu)(\rho_f^*-\rho_c)} \sim L^* \frac{\Delta}{(\rho_f^*-\rho_c)}.$$

The form of such a wide cluster is shown in Fig. 10(a); its dimension is

$$L = L_{\mathrm{rd}} \frac{\overline{\rho}-\rho_c}{\Delta} \tag{34}$$

and the region of the mean car density corresponding to this limit is specified by the inequality

$$\frac{\rho^*-\rho_c}{\Delta} \gg \frac{L^*}{L_{\mathrm{rd}}} \frac{\Delta}{(\rho_f^*-\rho_c)}. \tag{35}$$

The resulting dependence of the cluster dimension on the mean car density $\overline{\rho}$ is illustrated in Fig. 10(b).

IV. SYNCHRONIZED MODE ↔ JAM PHASE TRANSITION: BRIEF DISCUSSION

In Sec. II we have considered the phase transition between the free flow and the synchronized mode. However, according to the experimental data [12] there is an additional phase transition in traffic flow regarded as the transition between the synchronized motion and the jammed ''stop-and-go'' traffic. This transition occurs at extremely high vehicle densities ρ coming close to the limit value ρ_0.

The present section briefly demonstrates that the developed model for the driver behavior also predicts a similar phase transition at high car densities. To avoid possible misunderstandings we, beforehand, point out that the model in its present form cannot describe details of the synchronized mode ↔ jam transition because we have not taken into account the delay in the driver response to variations in headway. The latter is responsible for the formation of the stop-and-go pattern, so to describe the jammed traffic on multilane highways we at least should combine a governing equation for the order parameter h and a continuity equation similar to Eqs. (20) and (19) with an equation for the car velocity relaxation similar to Eq. (2). This question, however, is worthy of individual study. Besides, the approximations used in Sec. III to characterize the synchronized mode at the car densities near ρ_c do not hold here.

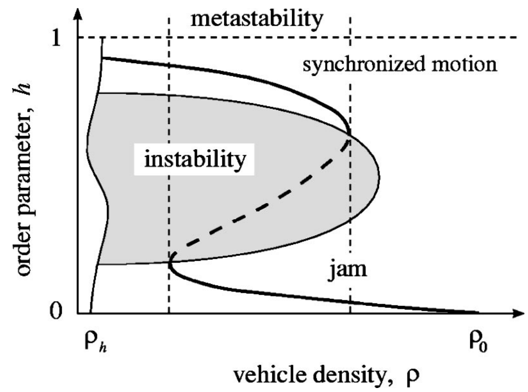

FIG. 11. The instability region and the $h(\rho)$ dependence describing the transition from the synchronized (congested) phase to the heavy congested phase (a jam) in the region of high car density.

In Sec. II we have studied the dependence of the order parameter h on the car density ignoring the first term on the right-hand side of Eq. (17) caused by the danger of lane changing. This assumption is justified when the car density is not too high. In extremely dense traffic flow, when the car density exceeds a certain value, $\rho > \rho_h \lesssim \rho_0$, changing lanes becomes sufficiently dangerous and the function $\Phi(h,v,\rho)$ describing the driver behavior is to depend strongly on the vehicle density in this region. In addition, the vehicle motion becomes sufficiently slow. Under such conditions the former term on the right-hand side of expression (17) should be dominant and, thus, $\partial\phi/\partial\rho > 0$. Therefore, the stable vehicle motion corresponding to $\partial\phi/\partial h > 0$ matches the decreasing dependence of the order parameter $h(\rho)$ on the vehicle density ρ for $\rho > \rho_h$. So, as the vehicle density ρ increases the curve $h(\rho)$ can again go into the instability region (in the $h\rho$ plane), which has to give rise to a jump from the synchronized mode with greater values of the order parameter to a new traffic state with its less values (Fig. 11). Obviously, this transition between the two congested phases also exhibits the same hysteresis as the one described in Sec. II.

We identify the latter traffic state with the jammed vehicle motion. Indeed, in extremely dense traffic lane changing is practically depressed, making the car ensembles at different lanes independent of one another. So, in this case vehicle flow has to exhibit weak multilane correlations and we should ascribe to it small values of the order parameter h. It should be noted that the experimental single-vehicle data [33] demonstrates strong correlations of variations in the traffic flux and the car density for both the free flow and the stop-and-go motion. By contrast, the synchronized mode is characterized by small values of the cross-covariance between flow, speed, and density. In other words, for the free flow and the stop-and-go motion the traffic flux $j = \vartheta\rho$ should depend directly on the car density ρ, as it must in the present model if we set $h = 0$.

Finalizing the present section we point out that the given model treats the jammed phase as a ''faster'' vehicle motion than the synchronized mode at the *same* values of the order parameter. There is no contradiction with the usual view on the synchronized mode as a high flux traffic state. The latter corresponds to the traffic flow at the vehicle densities near the free flow ↔ synchronized mode phase transition rather than close to the limit value ρ_0. Besides, an ordinary driver's experience prompts that a highly dense traffic flow can be blocked at all if one of the cars begin to change lanes. Nevertheless, in order to describe, at least qualitatively, the real features of the synchronized mode $\leftrightarrow$ stop-and-go waves phase transition a more sophisticated model is required. The present description only relates it to the instability of the order parameter at high values of the vehicle density.

Besides, the present analysis demonstrates also the nonmonotonic behavior of the order parameter as the car density increases even if we ignore the hysteresis regions and focus our attention on the stable vehicle flow regions only. It should be noted that a similar nonmonotonic dependence of the lane change frequency on the car density as well as the platoon formation has been found in the cellular automaton model for two-lane traffic [44].

V. CLOSING REMARKS

To conclude this paper we recall the key points of the developed model. We have proposed an original macroscopic approach to the description of multilane traffic flow based on an extended collection of the traffic flow state variables. Namely, in addition to such characteristics as the car density ρ and mean velocity v being actually the zeroth and first moments of the ''one-particle'' distribution function, we introduce a new variable h called the ''order parameter.'' It stands for the *internal* correlations in the car motion along different lanes that are due to lane-changing maneuvers. The order parameter, in fact, allows for the essentially ''many-body'' effects in the car interaction so it is treated as an independent-state variable.

Taking into account the general properties of the driver behavior we have stated a governing equation for the order parameter. Based on current experimental data [11–14,33] we have assumed the correlations in the car motion on multilane highways to be due to a small group of ''fast'' drivers, i.e. the drivers who move substantially faster than the statistically mean vehicle continuously overtaking other cars. These ''fast'' cars, on one hand, increase individually the total rate of vehicle flow but, on the other hand, make the accident danger greater and, thus, cause the statistically mean driver to decrease the velocity. The competition of the two effects depends on the car density and the mean velocity and, as shown, can give rise to the traffic flow instability. It turns out that the resulting dependence of the order parameter on the car density describes in the same way the experimentally observed sequence of free flow $\leftrightarrow$ synchronized motion $\leftrightarrow$jam phase transitions typical for traffic flow on highways [12]. Besides, we have shown that both these transitions should be of the first-order type and exhibit hysteresis, matching the experimental data [12–14]. The synchronized mode is characterized by a large value of the order parameter, whereas the free flow and the jam match its small values. The latter feature enables us to treat the jam as a phase comprising the vehicle flows at different lanes with weak mutual interaction because of the lane changing being depressed.

In order to illustrate the characteristic features of the car clusters that self-organize under these conditions we have considered a simple model that deals only with the evolution of the car density and the order parameter. In particular, it is shown that in the steady state the car density inside the cluster and the free flow being in equilibrium with the cluster, as well as the velocity at which the cluster moves upstream, are fixed and determined by the basic properties of the traffic flow. On the contrary, the size of the car cluster depends on the initial conditions.

Finally, we would like to underline that the developed model takes into account only one effect that causes the traffic flow instability. The other, the delay in the driver control over the headway, seems to be responsible for the stop-and-go waves in the jammed phase (for a review of the continuum description of this phenomena see, e.g., [24,25]). So, combining the two approaches into one model it enables a detailed description of a wide class of phenomena occurring in the transitions from free flow to the heavy congested phase on highways. In this way the order parameter model could also describe the formation of a local jam on a highway whose boundaries comprise both of the phase transitions. In the present form it fails to do this because the free flow and the jammed traffic are characterized by small values of the order parameter.

Concerning a possible derivation of the order-parameter model from the gas-kinetic theory we note that the appearance of the fast driver platoons demonstrates a substantial deviation of the car distribution function from the monotonic quasiequilibrium form. So, to construct an adequate system of equations dealing with the moments of the distribution function a more sophisticated approximation is required.

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